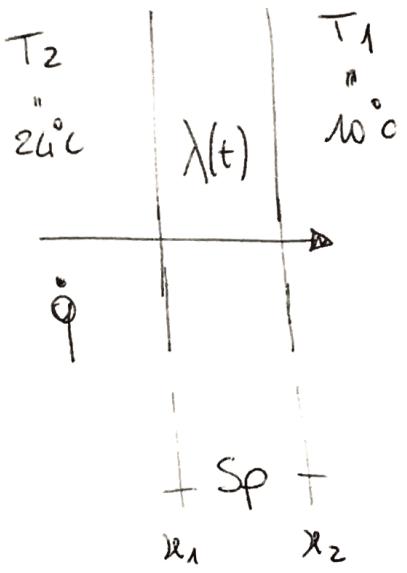
21/02/2018

ESERCIZIO



$Sp = 1\,cm$

$T_1 = 10°C$

$T_2 = 24°C$

$\lambda(t) = 0{,}035 + 0{,}0002\,t \left[\frac{W}{m°C}\right]$ $\rightsquigarrow$ $\lambda(t) = (a + bt)$

$$\dot{q} = \frac{\dot{Q}}{S} = -\lambda\, \text{grad}\, T \xrightarrow{\text{Caso monodimensionale}} \frac{\dot{Q}}{S} = -\lambda(t)\frac{dt}{dx}$$

$$\frac{\dot{Q}}{S}dx = -\lambda(t)\,dt \xrightarrow{\int} \int_{x_1}^{x_2} \frac{\dot{Q}}{S}dx = -\int_{t_1}^{t_2} \lambda(t)\,dt$$

Costante

$$\dot{q}\,x\Big|_{x_1}^{x_2} = -a\,t\Big|_{t_1}^{t_2} - b\,\frac{t^2}{2}\Big|_{t_1}^{t_2}$$

$t_1 = T_2$

$t_2 = T_1$

$x_1 = 0$

$x_2 = x$

$$\dot{q}\,x\Big|_{0}^{x} = -a\,t\Big|_{T_2}^{t(x)} - b\,\frac{t^2}{2}\Big|_{T_2}^{t(x)}$$

in $x \Rightarrow t(x)$

$x = x_1 = 0 \rightarrow T_2$

$x = x_2 = Sp \rightarrow T_1$

$x \rightarrow t(x)$

$$\dot{q}\,x = -\left[a\left(t(x) - T_2\right)\right] - \frac{b}{2}\left(t^2(x) - T_2^2\right)$$

$$\dot{q}\,x = aT_2 - a\,t(x) + \frac{b}{2}T_2^2 - \frac{b}{2}t^2(x)$$

$$* \quad \boxed{\dot{q}\,x = -\frac{b}{2}t^2(x) - a\,t(x) + aT_2 + \frac{b}{2}T_2^2}$$

trovo il flusso usando *

$$\dot{q}(sp) = \frac{b}{2}\left[T_2^2 - T_1^2\right] + a\left[T_2 - T_1\right]$$

$$\dot{q}\,10^{-2} = 10^{-4}\left(24^2 - 10^2\right) + 0{,}035\,(24-10) \Rightarrow \dot{q} = 4{,}76 + 49 = 53{,}8 \ W/m^2$$

$$\boxed{\dot{q} = 53{,}8 \ W/m^2}$$

$$x = \frac{3}{4}\,sp = 0{,}75\ cm = C$$

$$* \quad \dot{q}\cdot C = -\frac{b}{2}t^2(x)\Big|_c - a\,t(x)\Big|_c + aT_2 + \frac{b}{2}T_2^2$$

$$\frac{b}{2}t^2(x)\Big|_c + a\,t(x)\Big|_c - aT_2 - \frac{b}{2}T_2^2 + \dot{q}C = 0$$

eq di 2° grado

$$\frac{b}{2}t^2(x)\Big|_c + a\,t(x)\Big|_c + [\ \ ] = 0$$

$$10^{-4}\,t^2(x) + 35\,10^{-3}\,t(x) - 0{,}49 = 0$$

$$t(x)\Big|_{c\,1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t^2(x) + 350\,t(x) - 4900 = 0$$

$a = 1$ $\quad b = 350$ $\quad C = 4900$

$$t(x)\Big|_{c\,1,2} = \frac{-350 \pm 377}{2} = \begin{cases} \not{0} \\ 13{,}5\ ^\circ C \end{cases}$$

temperatura a $\frac{3}{4}$ dello spessore $\boxed{t = 13{,}5\ ^\circ C}$